# Mining Your Ps and Qs: Detection of Widespread Weak Keys in Network Devices

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# Intro

- Detecting weak keys/signatures in the wild
- Collected public keys/certificates

- Tried to figure out how weak keys/signatures were generated

# Public keys and Randomness

- Public key cryptography used everywhere!
    - TLS (used in HTTPS), SSH, …
- Based on *randomly* generated secret keys

# Public keys and Randomness

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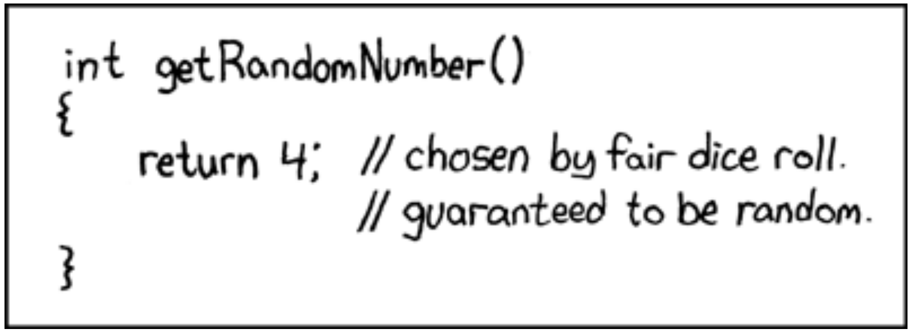
  - TLS (used in HTTPS), SSH, …
- Based on *randomly* generated secret keys
- What if they are not random?




from: xkcd (https://www.explainxkcd.com/wiki/index.php/221:_Random_Number)

# Collecting public keys

**Finding Hosts**
Nmap from EC2
25 hosts, ~25 hours

| Port 443 (HTTPS) | Port 22 (SSH) |
|---|---|
| 29 million hosts | 23 million hosts |

**Retrieving Keys**
Event Driven Process
3 hosts, <48 hours

| Port 443 (HTTPS) | Port 22 (SSH) |
|---|---|
| 13 million hosts | 10 million hosts |

**Parsing Certs**
OpenSSL, database

| Certificates |
|---|
| 6 million certificates<br>(2 million browser-trusted) |

# What could go wrong?

1. Repeated keys

# Repeated keys

| | SSL Observatory (12/2010) | Our TLS scan (10/2011) | Our SSH scans (2-4/2012) |
|---|---|---|---|
| Hosts with open port 443 or 22 | ≈16,200,000 | 28,923,800 | 23,237,081 |
| Completed protocol handshakes | 7,704,837 | 12,828,613 | 10,216,363 |
| Distinct RSA public keys | 3,933,366 | 5,656,519 | 3,821,639 |
| Distinct DSA public keys | 1,906 | 6,241 | 2,789,662 |
| Distinct TLS certificates | 4,021,766 | 5,847,957 | — |
| Trusted by major browsers | 1,455,391 | 1,956,267 | — |

- TLS: 7,770,232 hosts (61%)
- SSH: 6,642,222 hosts (65%)

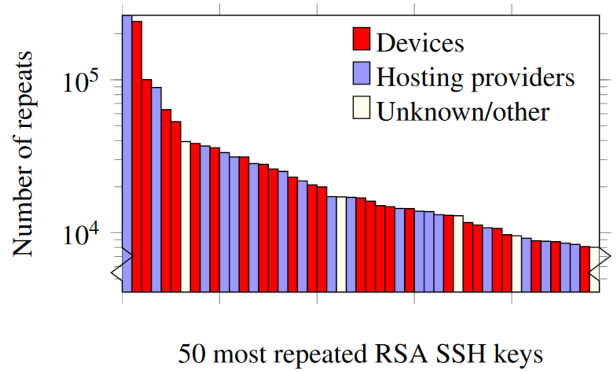
# Shared keys

**Non-vulnerable reasons for shared keys**

- Corporations shared keys across certificates
- Shared hosting providers

**Vulnerable reasons for shared keys**

- Default certificates and keys
- Low entropy problems



# What could go wrong?

1. Repeated keys

2. Repeated factors in RSA keys

# RSA revisited

- Generate two random prime numbers $p, q$

- Public key: $(e, N)$, $N = pq$ (Usually $e = 65537$)
- Private key: $d = e^{-1} \pmod{\phi}, \phi = (p-1)(q-1)$

- Why is it difficult to break?

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- Why is it difficult to break?
  1. Hard to factorize $N$, so difficult to get $\phi$ and calculate $d$
  2. For given encrypted message $m^e \pmod{N}$, it's hard to recover $m$ (DLP)

# Repeated factors

- What if $N_1 = pq, N_2 = pr$?
  The greatest common divisor (GCD) is $p$.
- Euclidean method! (from 300 BC)
  - Takes 15μs for two 1024-bit numbers

- For multiple $N$s, Bernstein's algorithm can be used.

https://www.worldhistory.org/image/4139/euclid-of-alexandria/

# Result?

- 11,170,883 RSA keys
- 1.3 hours on EC2 Cluster Compute Eight Extra Large Instance
  - only $5!
- Got 2,134 prime factors
- Computed private keys for 64,081 TLS hosts (0.50%)


https://i.insider.com/5c7967b3eb3ce8763f505bf5?width=700&format=jpeg&auto=webp

# What could go wrong?

1. Repeated keys

2. Repeated factors in RSA keys

3. Repeated DSA signature randomness

# DSA revisit

- Pick two random prime numbers: $p, q$
- Private key: $x$ / Public key: $y = g^x \bmod p$

- Signature $(r, s)$:

$$\text{For random nonce } k\text{:}$$
$$r = \left(g^k \bmod p\right) \bmod q$$
$$s = k^{-1}(H(m) + xr) \bmod q$$

# Ephemeral key is shared

$$r = \left(g^{k} \bmod p\right) \bmod q$$
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$$s_1^{-1}(H(m_1) + xr) = k = s_2^{-1}(H(m_2) + xr) \pmod q$$

# Result?

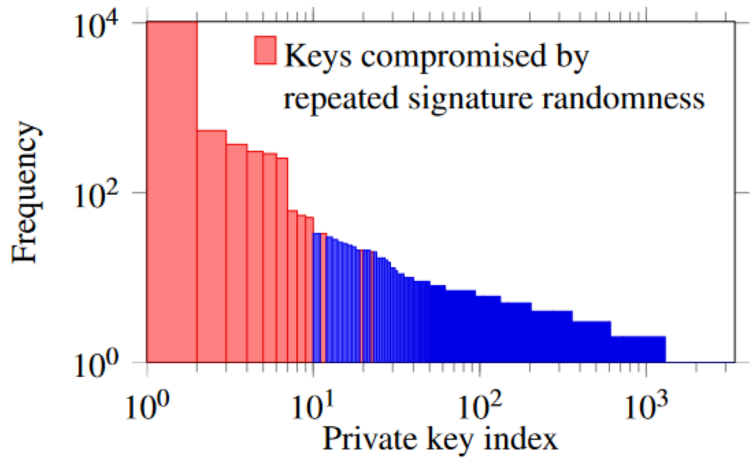
- 9,114,925 DSA signatures from SSH

- 4,094 signatures with same public key and $r$
- Recovered 281 distinct private keys
- These keys are used in 105,728 hosts (1.6%)

# Result?

- Clustered vulnerable signatures by $r$ values manufacturers

- 75.8% of the cases were from two manufacturers

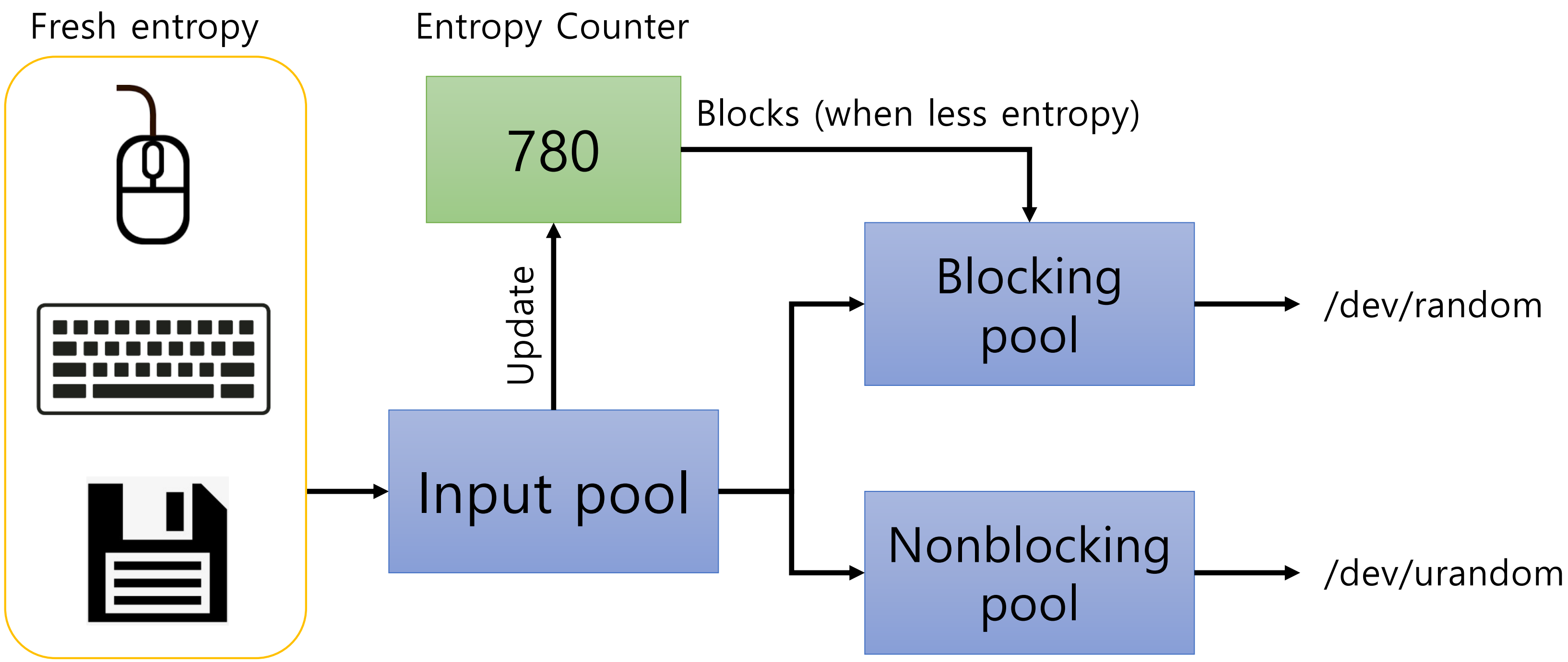


# Final result

| | Our TLS Scan | | Our SSH Scans | |
|---|---|---|---|---|
| Number of live hosts | 12,828,613 | (100.00%) | 10,216,363 | (100.00%) |
| ...using repeated keys | 7,770,232 | (60.50%) | 6,642,222 | (65.00%) |
| ...using vulnerable repeated keys | 714,243 | (5.57%) | 981,166 | (9.60%) |
| ...using default certificates or default keys | 670,391 | (5.23%) | | |
| ...using low-entropy repeated keys | 43,852 | (0.34%) | | |
| ...using RSA keys we could factor | 64,081 | (0.50%) | 2,459 | (0.03%) |
| ...using DSA keys we could compromise | | | 105,728 | (1.03%) |
| ...using Debian weak keys | 4,147 | (0.03%) | 53,141 | (0.52%) |
| ...using 512-bit RSA keys | 123,038 | (0.96%) | 8,459 | (0.08%) |
| ...identified as a vulnerable device model | 985,031 | (7.68%) | 1,070,522 | (10.48%) |
| ...model using low-entropy repeated keys | 314,640 | (2.45%) | | |

# Weak entropy and the Linux RNG

- Nearly everything uses /dev/urandom



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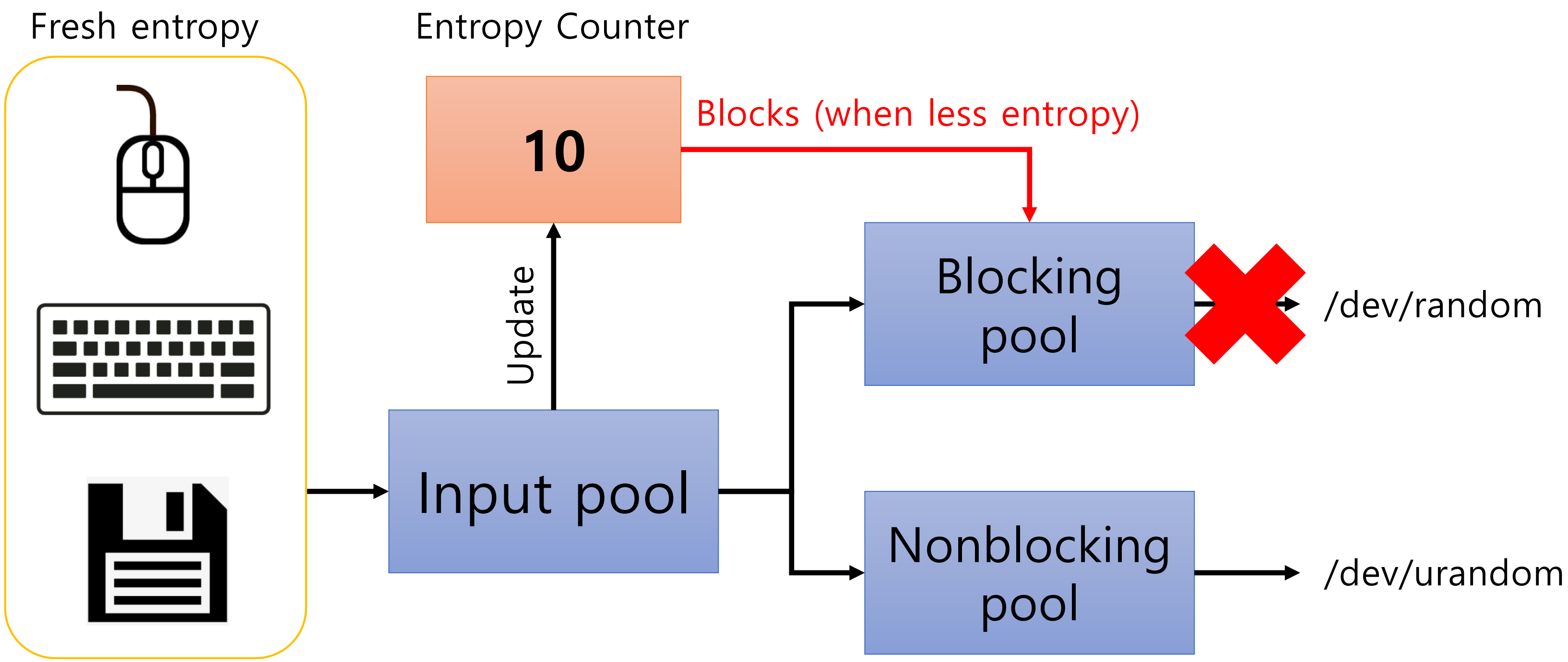

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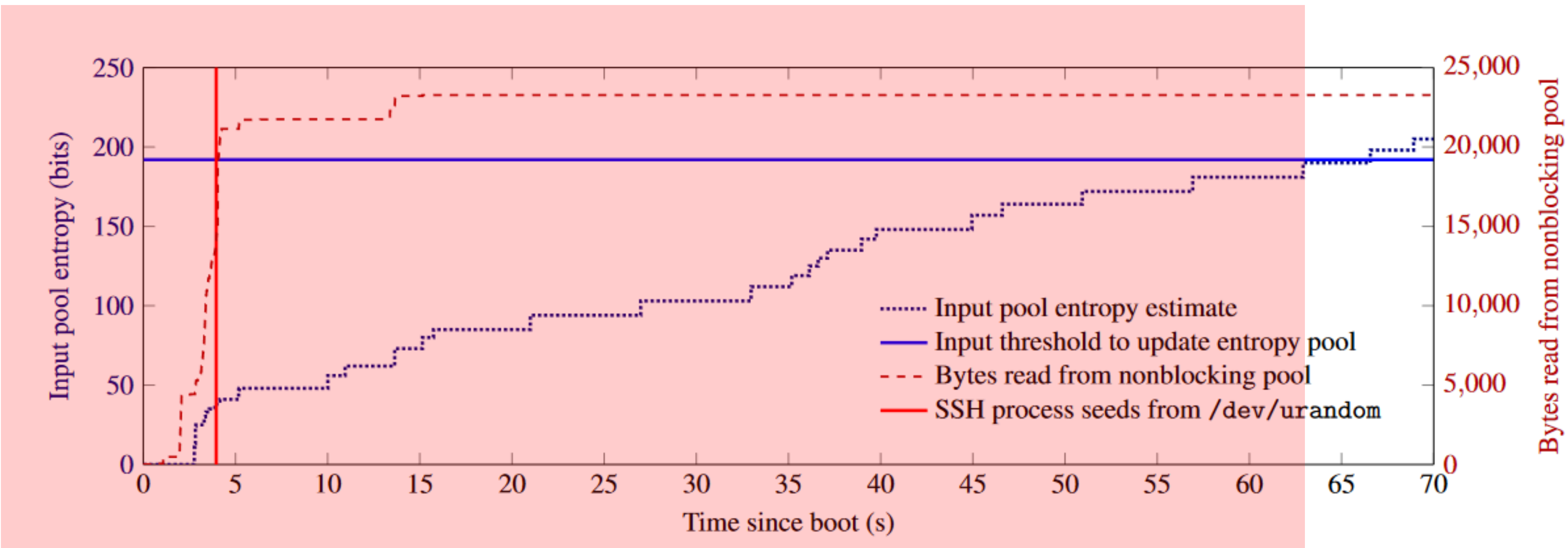
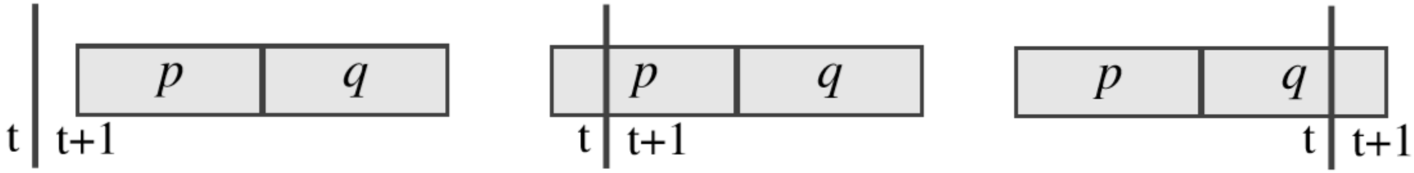



Ubuntu 10.04 test system (typical boot)

# Factorable RSA keys




Both factor repeated
Not bad

Both factor different
Very good

Only one different
Problematic!

# Defense

- **Lessons** for OS developers, crypto library developers, app developers, device makers, certificate authorities, end users, security and crypto researchers
- **More entropy sources**
  - Add hardware sources
  - Kernel collects more aggressively
- **Better communication between applications and OS**
  - /dev/urandom isn’t providing the service people need
- **Create public key check service for end users**

# Conclusion

- Studied entropy via global perspective on public keys

- Found widespread vulnerabilities
  - Shared keys (5.6% of TLS hosts; 9.6% of SSH)
  - Factorable RSA keys (0.5% of TLS hosts; 0.03% of SSH)
  - Repeated DSA randomness (1.0% of SSH hosts)

- Secure random number generation is still difficult

# Related works

**Problems with random number generation**

- “Randomness and the Netscape browser”, Dr. Dobb’s Journal 21 (1996)
- DSA-1571-1 OpenSSL—Predictable random number generator (2008)
- “Analysis of the Linux random number generator”, SP ’06

**Weak entropy and cryptography**

- Console hacking 2010: PS3 epic fail, Talk at 27th Chaos Communication Congress (2010)
- “When good randomness goes bad: Virtual machine reset vulnerabilities and hedging deployed cryptography”, NDSS ‘10

# Follow-up works

## Other cryptographic vulnerabilities

- Unsecure ECDSA key
  "Elliptic curve cryptography in practice.", FC '14
- Diffie-Hellman algorithm
  "Imperfect Forward Secrecy: How Diffie-Hellman Fails in Practice", CCS '15

## Malfunction of RNG

- "Security Analysis of Pseudo-Random Number Generators with Input: /dev/random is not Robust", CCS '13
- "Not-so-random numbers in virtualized Linux and the Whirlwind RNG", SP '14

# Good Questions

- How would detecting weak entropy be possible in the crypto graphic primitives level?
- Is there any other way to provide almost "perfect" randomnes s, without using hardware RNG or factory adding  entropy?
- If insufficient entropy is the issue, why don't we use another s oftware package that blocks until enough entropy is acquired for security-critical hosts?
- Would RSA really be a problem if the same key is generated?

# Best Questions

**Jaehyun:** if the keys itself are not safe regardless of how we generate the keys with much care, what kind of countermeasures should be made?

**A:** This paper is not about PRNG, and there are various PRNGs which is not able to compare.

**Q:** Is any new operation added to make randomness in OS?

**A:** /dev/urandom is not a problem, what do you mean?